DYNAMIC MODEL REDUCTION USING MODAL TRUNCATION IN THE BUILDING MOTION PROBLEM

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ABSTRACT

Modal truncation, an advanced algorithm for model reduction in dynamic systems, efficiently simplifies complex models by selectively discarding less influential eigenmodes, maintaining a balance between computational efficiency and model accuracy. This paper explores the algorithm's application to a 48th order building model. Proceed to reduce this model to lower orders, then analyze errors in time and frequency domains. Modal truncation algorithm systematically reduces model dimensions while preserving critical dynamic attributes. Numerical simulations reveal a favorable reduction order range (from order 6th to order 25th) for optimal balance, with sensitivity observed at order 25th. From the results obtained, depending on specific requirements, users can use a lower-order model corresponding to the allowed error to replace the original system. Recommendations include iterative refinement for adaptive reduction orders and in-depth analysis around critical points. This algorithm becomes an effective method for researchers dealing with high-dimensional dynamic systems, offering simpler yet accurate model representations. As technology develops, continued refinements and applications of modal truncation are expected, solidifying its role in the realm of model reduction.

Keywords: Modal truncation; model reduction; dynamic systems; eigenmode analysis; duilding motion model.

1. Introduction

Modal truncation is an advanced algorithm used for model reduction in dynamic systems. It efficiently simplifies complex models by selectively discarding less influential eigenmodes, maintaining a balance between computational efficiency and model accuracy. The algorithm's mathematical approach involves eigenvalue analysis to identify optimal truncation points, ensuring that the reduced model retains essential system characteristics. Widely applied in disciplines like control systems and fluid mechanics, modal truncation offers improved computational efficiency and reduced memory requirements, making it a valuable tool for researchers dealing with high-dimensional dynamic systems [1].

The modal truncation algorithm demonstrates its applicability in various applications with suitable enhancements for specific targets. Notable include instances: [2] addresses the modal truncation problem in nonviscously damped systems, and employ the Neumann expansion theorem and frequency shifting technique to calculate steady-state responses accurately. The Generalized Mode Acceleration Method (GMAM) and Modal Truncation Augmentation Method (MTAM) are introduced to handle modal truncation, overcoming issues with the stiffness matrix and addressing equilibrium equations. [4] compares three algorithms for dynamic model reduction of power systems, including modal truncation. The reduced model accelerates the analysis and control of a power system by maintaining dominant oscillation modes. The comparison is conducted on a dynamic 39-bus New England test system. [5] introduces a systematically controlled modal truncation strategy for evaluating the response function of the structural pounding problem, especially in cases of high nonlinearity and nonsmoothness of contact problems. The controlled modal truncation strategy shows improved computational accuracy compared to the classical technique. [5] investigates the effect of modal truncation in experimental modal analysis, revealing that neglecting higher modes introduces errors in system response. The study emphasizes the dependence of system response on frequency and load distribution. [6] focuses on limiting extreme values in frequency response functions, citing a constraint related to eigenfrequency maximization. Using modal truncation and modal truncation augmentation in reduced-order models ensures computational efficiency while maintaining accuracy. [7] revisits modal truncation from an optimization perspective, formulating the concept of pole dominances as a solution to the optimal modal truncation problem. Numerical examples highlight the concept and optimization approach. Researchers in [8] propose a method to determine the number of truncated modes for the mode superposition method based on the hysteretic damping model. Indexes such as cumulative modal contribution coefficients are utilized for computational accuracy considerations. Some other applications can also be mentioned in [9-11] or other related documents. Furthermore, to leverage the efficient model reduction capabilities of modal truncation, the author applies this algorithm to simplify a building model of an order 48th in [12], subsequently analyzing corresponding errors in the time and frequency domains. The findings provide insights into the effectiveness of this approach for the considered object.

2. Materials and methods

Modal truncation operates by focusing on the eigenmodes of a dynamic system, recognizing their pivotal role in characterizing system behavior. By strategically truncating modes based on their influence, the algorithm strikes a balance between computational efficiency and model accuracy. This involves a meticulous eigenvalue analysis to identify and discard less impactful modes, ensuring a concise yet representative model. The algorithm's mathematical formulation is rigorous, employing eigenvalue analysis to determine optimal truncation points. Researchers leverage this formulation to judiciously select thresholds, guaranteeing that the reduced model retains essential system characteristics [1]. The main content of this algorithm is described in the flow chart in Figure 1, including the following steps:

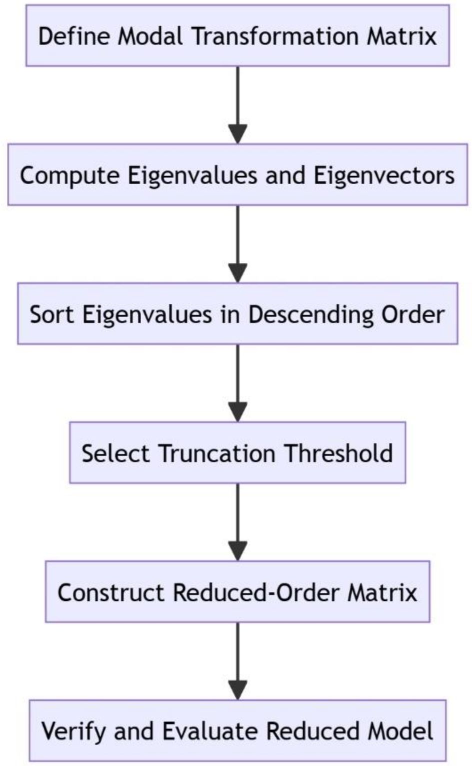

Figure 1: *Modal truncation algorithm flow chart*

- Step 1: Initiate the algorithm by defining the modal transformation matrix, often derived from the system's dynamic equations. This matrix encapsulates the evolution of system modes over time.

- Step 2: Utilize the modal transformation matrix to compute the eigenvalues and corresponding eigenvectors, representing the dynamic characteristics of the system. These values elucidate the system's inherent modes.

- Step 3: Arrange the eigenvalues in descending order of magnitude. Modes associated with larger eigenvalues encapsulate more dynamic information and are typically retained during the model reduction process.

- Step 4: Establish a truncation threshold, denoting the minimum absolute value for eigenvalues. This threshold delineates which modes should be preserved and which can be disregarded, facilitating an optimal reduction.

- Step 5: Based on the selected eigenvalues, craft a reduced-order matrix. This matrix, smaller in size compared to the original, retains essential system characteristics while significantly reducing computational complexity.

- Step 6: Validate the reduced-order model by comparing its outputs with those of the original model. Assess the degree of discrepancy to ascertain the accuracy and appropriateness of the reduction for the intended application.

The modal truncation algorithm, through these systematic steps, furnishes a reduced-order version of the dynamic model, effectively minimizing model dimensions while preserving critical dynamic attributes of the system. This approach is indispensable for researchers and practitioners seeking efficient yet accurate representations of complex dynamic systems.

3. Results and discussion

The motion problem in a building is characterized by the following dynamic equations:

$$\begin{aligned}\dot{\mathbf{x}}(t) &= \mathbf{Ax}(t) + \mathbf{Bu}(t) \\ \mathbf{y}(t) &= \mathbf{Cx}(t)\end{aligned} \tag{1}$$

where: **x**(t) represents the state vector describing the motion of the building; **u**(t) is the input vector representing external forces or inputs affecting the building's motion; **y**(t) is the output vector representing the observed displacement or response of the building; **A** is a 48x48 matrix describing the system dynamics; **B** is a 48x1 matrix representing the input matrix; **C** is a 1x48 matrix defining the output matrix.

This system captures the motion dynamics of a multi-story building, commonly encountered during seismic events such as earthquakes. The motion problem in a building serves as a benchmark example, particularly within the context of the SLICOT benchmark examples for model reduction. This benchmark aids in the development and evaluation of model reduction techniques, crucial for efficiently simulating and analyzing the behavior of complex dynamic systems like multi-story buildings subjected to external forces, the parameters of matrices **A**, **B**, and **C** are provided in [12].

The implementation of the modal truncation algorithm and the subsequent Matlab program execution involves reducing the system order from order 48 to order 1 [9]. For each order reduction, the maximum errors are computed, specifically the maximum absolute error (Hinf norm) between the original system of order n and the reduced system of order r across the entire time domain, as illustrated in Figure 2. Additionally, the maximum relative error (Hinf norm) between the original system of order n and the reduced system of order r is calculated across the entire frequency domain, depicted in Figure 3.

In the conducted numerical experiment, the modal truncation algorithm was applied to a linear time-invariant dynamical system with varying reduced orders (r) ranging from 1 to the size of the system (n). The results reveal interesting insights into the relationship between the system order reduction and the corresponding errors.

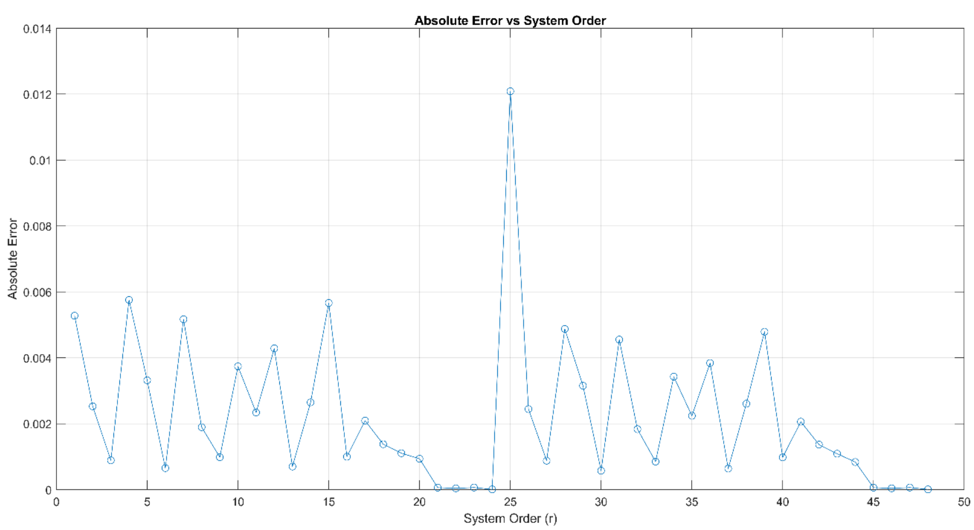

Figure 2: *The absolute error corresponding to the system is reduced in order*

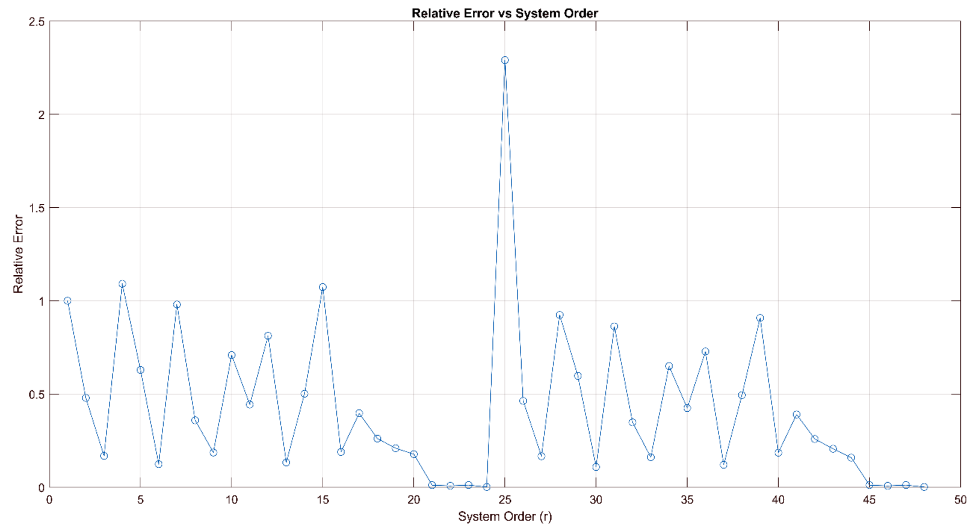

Figure 3: *The relative error corresponding to the system is reduced in order*

For each reduction order (*r*), absolute error and relative error were computed and analyzed. With the results obtained from the simulation, we have the following assessments:

- Error analysis: The computed absolute errors range from approximately $7.743311x10^{-6}$ to $1.208590x10^{-2}$, indicating a relatively small range. This suggests that the reduced-order models generally exhibit low absolute discrepancies from the original system.

Relative errors, ranging from around $1.467555x10^{-3}$ to *2.290587*, reflect variations in the proportional error relative to the norm of the original system. The significant spike at $r = 25$ warrants closer scrutiny.

- Optimal reduction order: The reduction orders between 6 and 25 appear to offer a favorable balance between computational efficiency and model accuracy, as indicated by lower absolute and relative errors. Beyond this range, there is a discernible increase in errors, emphasizing the importance of a judicious selection of the reduction order.

Figures 4 and 5 illustrate the frequency response and time response between the reduced order system (6th order and 25th order) compared to the original system, respectively. From this simulation result, we see that the 6th-order reduction system has better quality than the 25th-order reduction system. The responses of the 6th order reduction system closely follow the original system while the 25th order reduction system is very different from the original system.

- Sensitivity and thresholds: The sudden increase in errors at $r = 25$ suggests a potential sensitivity of the model to the chosen reduction order. Further investigations into the system's eigenvalue distribution and sensitivity analysis may reveal insights into such critical points.

Recommendations:

+ Iterative refinement: Implementing an iterative refinement strategy, where the reduction order is adjusted based on observed error trends, could offer an adaptive approach to maintaining accuracy while minimizing computational costs.

+ In-depth analysis: Conducting a more granular analysis around critical points, such as *r = 25*, may unveil the specific system characteristics leading to increased errors.

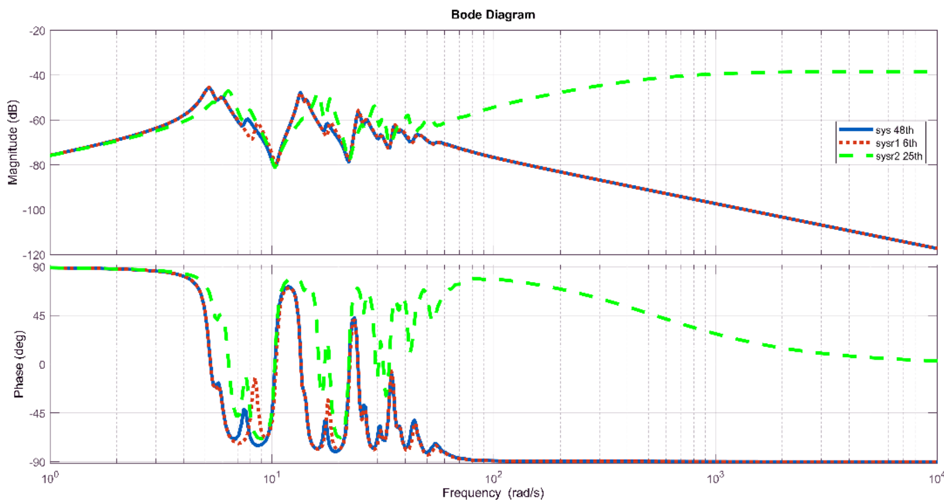

Figure 4: *The frequency response of the original system and the reduced order system*

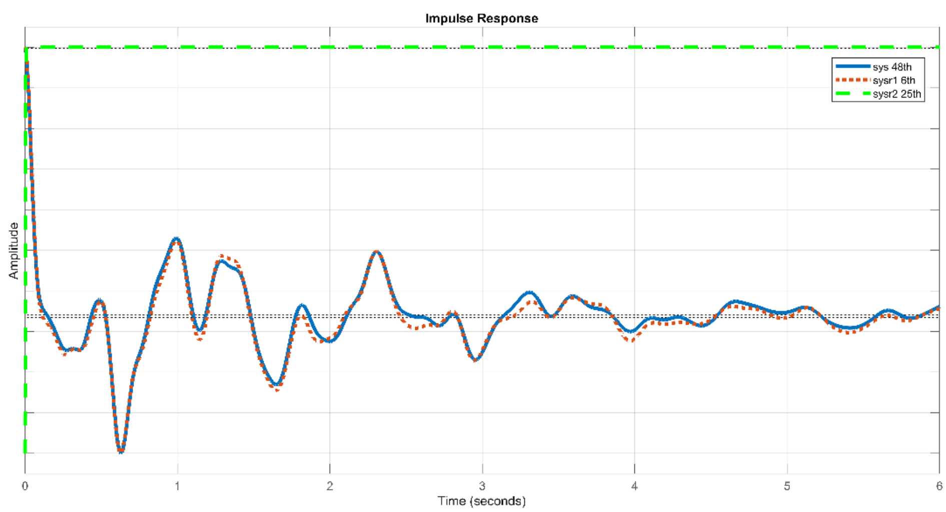

Figure 5: *The impulse response of the original system and the reduced order system*

4. Conclusion

Modal truncation proves its effectiveness in model reduction, offering a crucial balance between computational efficiency and model accuracy. The practical experiment involving the reduction of a complex dynamic system, exemplified by the motion problem in a building, yields valuable insights. The analysis of errors emphasizes a favorable reduction order range (6 to 25) that achieves an optimal balance. However, the sensitivity observed at *r = 25* underscores the need for further investigation into system characteristics.

In conclusion, modal truncation stands as a valuable asset for researchers and practitioners dealing with high-dimensional dynamic systems. Its systematic reduction steps provide efficient yet accurate representations. As technology advances, continued refinements and applications of modal truncation are expected, solidifying its role in the realm of model reduction.

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GIẢM BẬC MÔ HÌNH ĐỘNG HỌC SỬ DỤNG CẮT NGẮN PHƯƠNG THỨC TRONG BÀI TOÁN CHUYỂN ĐỘNG TÒA NHÀ

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Ngày nhận bài 08/01/2024, ngày nhận đăng 13/3/2024

Cắt ngắn phương thức, một thuật toán nâng cao để giảm mô hình trong các hệ thống động học, đơn giản hóa một cách hiệu quả các mô hình phức tạp bằng cách loại bỏ có chọn lọc các giá trị riêng ít ảnh hưởng, duy trì sự cân bằng giữa hiệu quả tính toán và độ chính xác của mô hình. Bài viết này khám phá ứng dụng của thuật toán vào mô hình toà nhà có bậc 48. Tiến hành giảm bậc mô hình này xuống các bậc thấp hơn, sau đó phân tích sai số trong miền thời gian và tần số. Thuật toán Cắt ngắn phương thức giảm kích thước một cách có hệ thống các mô hình trong khi vẫn bảo toàn các thuộc tính động học quan trọng. Các mô phỏng số cho thấy phạm vi giảm bậc thích hợp (từ bậc 6 đến bậc 25) để có sự cân bằng tối ưu, với độ nhạy được quan sát ở bậc thứ 25. Từ kết quả thu được, tùy theo yêu cầu cụ thể, người dùng có thể sử dụng mô hình bậc thấp hơn tương ứng với sai số cho phép để thay thế hệ thống ban đầu. Các khuyến nghị bao gồm sàng lọc lặp lại cho các bậc được giảm thích ứng và phân tích chuyên sâu xung quanh các điểm tới hạn. Thuật toán này trở thành một phương pháp hiệu quả cho các nhà nghiên cứu xử lý các hệ thống động học nhiều chiều, đưa ra các biểu diễn mô hình đơn giản hơn nhưng chính xác. Khi công nghệ phát triển, dự kiến sẽ tiếp tục có những cải tiến và ứng dụng của Cắt ngắn phương thức, củng cố vai trò của nó trong lĩnh vực rút gọn mô hình.

Từ khóa: Cắt ngắn phương thức, giảm bậc mô hình; hệ thống động học; phân tích giá trị riêng; mô hình chuyển động tòa nhà.